2.

DISSERTATIO ACADEMICA;

DE MOTU CORPORUM LIBERO IN MEDIO RESISTENTE;

CUJUS PARTEM SECUNDAM,

VENIA AMPLISS. FACULTATIS PHILOSOPH. IN IMPERIALI ACADEMIA ABOËNSI,

PUBLICO EXAMINI MODESTE SUBJICIUNT

NATHANAËL GERH. AF SCHULTÉN,

Phil. Mag. Aboënsis,

&

JOHANNES CAROLUS EBELING,

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h. a. m. s.

ABOÆ, Typis FRENCKELLIANIS.

h. e., si substituatur in æquatione (9) valor ipsius Q, ex æquatione (8) depromtus,

$$Qndx.d^2y = [(xdz - zdx).d^2y - (xdy - ydx)d^2z].P \ldots (10)$$

$$\varphi\left(D, ds\sqrt{\frac{(xdy - ydx)P}{ndx.d^2y}}\right) =$$

$$\frac{\left(\frac{d^2s}{ds} - \frac{x.d^2y}{xdy - ydx}\right).\left(\frac{(xdy - ydx)Pds^2}{ndx\,d^2y}\right)}{ds} -$$

$$\frac{d.\left\{\frac{(xdy - ydx)Pds^2}{ndx.d^2y}\right\}}{2ds} \quad \ldots\ldots\ldots (11).$$

His quidem æquationibus trajectoria hoc in casu descripta definitur. Ad v & t quod attinet, substitutis valoribus ipsorum L & M in formulis generalibus supra inventis; habetur:

$$v^2 = \frac{(xdy - ydx)Pds^2}{ndx.d^2y};\ dt^2 = \frac{ndx.d^2y}{(xdy - ydx)P}.$$

Sit ex. gr. $R = \alpha Dv^2$ (pendente α ex massa atque superficie corporis moti), quæ hypothesis est rerum naturæ proxime congruens; sitque D quantitas constans; habebitur:

C ds . αD

$$ds \,.\, \alpha D \,.\, \frac{(xdy - ydx)\,Pds^2}{ndx\,.\,d^2y} = \left(\frac{d^2s}{ds} - \frac{xd^2y}{xdy - ydx}\right) \left(\frac{(xdy - ydx)\,Pds^2}{ndx\,.\,d^2y}\right) - \tfrac{1}{2}\,d\,.\left\{\frac{(xdy - ydx)\,Pds^2}{ndx\,.\,d^2y}\right\};$$

i. e., dividendo per: $\left(\frac{(xdy - ydx)\,Pds^2}{ndx\,.\,d^2y}\right)$, atque integrando:

$$\tfrac{1}{2}\,Log.\,\frac{1}{C} + \alpha Ds = Log.\left(\frac{ds}{xdy - ydx}\right) - \tfrac{1}{2}\,.\,Log.\left(\frac{(xdy - ydx)\,Pds^2}{ndx\,.\,d^2y}\right);$$ i. e.

$$\frac{1}{C}\,.\,e^{2\alpha Ds} = \frac{ndx\,.\,d^2y}{(xdy - ydx)^3\,.\,P},$$ vel:

$$P = \frac{Cndx\,.\,d^2y\,.\,e^{-2\alpha Ds}}{(xdy - ydx)^3}.$$

In hanc igitur æquationem, satis simplicem, hoc in casu æquatio (*11*) abiit. Quod si valorem ipsius P sic inventum in æquatione (*10*) substituas; hæc sequentem induet faciem:

$$Q = \frac{[(xdz - zdx)\,.\,d^2y - (xdy - ydx)\,.\,d^2z]\,.\,Ce^{-2\alpha Ds}}{(xdy - ydx)^3};$$

eodem-

eodemque prorsus modo, in casu præsente, inveniuntur:

$$v^2 = \frac{ds^2 . Ce^{-2\alpha Ds}}{(xdy - ydx)^2}, \quad dt^2 = \frac{1}{C}(xdy - ydx)^2 . e^{2\alpha Ds}.$$

§. V.

Casus æquationum (I) atque (II) speciales consideraturi, primum utique omnium, generalem, quam hæ dant æquationes, motuum in *vacuo* liberorum determinationem, breviter contemplemur. Posita igitur densitate medii $D = o$, observandum utique est, æquationem (I) eamdem omnino, ac fuit, manere; æquationem vero (II) eatenus immutari, ut jam sit: $\varphi\left(D, ds\sqrt{\frac{Ldy - Mdx}{dy . d^2x - dx . d^2y}}\right) = o.$

Habemus igitur, hoc in casu, æquationes:

$$L(dz . d^2y - dy . d^2z) + M(dx . d^2z - dz . d^2x) + N(dy . d^2x - dx . d^2y) = o \quad \ldots \quad (I'),$$

$$2(Ldx + Mdy + Ndz) = d . \left\{\frac{(Ldy - Mdx)ds^2}{dy . d^2x - dx . d^2y}\right\} \ldots (II');$$

quæ viam a corpore descriptam in genere determinant.

Ad velocitatem v atque tempus t quod attinet, valores eorum generales, in §. II allatos, hoc in casu nihil mutari, facile perspicitur.

Æquationem igitur (II') ad inferiorem statim gradum depressam habebis, si completa fuerit ($L\mathrm{d}x + M\mathrm{d}y + N\mathrm{d}z$) differentialis, i. e. si ea L, M, N inter habeatur relatio, ut sit:

$$\frac{\mathrm{d}L}{\mathrm{d}y} = \frac{\mathrm{d}M}{\mathrm{d}x},\ \frac{\mathrm{d}L}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}x},\ \frac{\mathrm{d}M}{\mathrm{d}z} = \frac{\mathrm{d}N}{\mathrm{d}y}.$$

De cetero, ut instituti rationem sequamur, æquationum nuper allatarum (I') & (II'), ad casus speciales, applicationes, hoc loco omittamus. Horum quosdam insigniores suo quemque loco in sequentibus adferre nobis visum, alios jamjam æquationum (I) & (II) casus contemplaturis, speciales illos quidem, at (ut infra præcipue apparebit) late tamen patentes.

Evanescat nempe aliqua potentiarum L, M, N, quas in præcedentibus omnes finita agentes vi posuimus. Fiat ex. gr. $L = o$; sitque brevitatis ergo elementum $\mathrm{d}x$ constans.

Æquatio igitur (I) in hanc abit:

$$M\mathrm{d}x . \mathrm{d}^2z - N\mathrm{d}x . \mathrm{d}^2y = o,\ \text{i. e.}\ M\mathrm{d}^2z - N\mathrm{d}^2y = o \,..\, (12).$$

Altera vero (II) sequentem induet formam:

$$\varphi\left(D, \mathrm{d}s\sqrt{\frac{M\mathrm{d}x}{\mathrm{d}x . \mathrm{d}^2y}}\right) = \frac{M\mathrm{d}y + N\mathrm{d}z}{\mathrm{d}s} - \frac{\mathrm{d}.\left(\frac{M\mathrm{d}x . \mathrm{d}s^2}{\mathrm{d}x . \mathrm{d}^2y}\right)}{2\mathrm{d}s};$$

h. e.

h. e.

$$\varphi\left(D, ds\sqrt{\frac{M}{d^2y}}\right) = \frac{Mdy + \frac{Md^2z}{d^2y}.dz}{ds} - \frac{d.\left(\frac{Mds^2}{d^2y}\right)}{2ds},$$

$$= \frac{Md^2s}{d^2y} - \frac{d.\left(\frac{Mds^2}{d^2y}\right)}{2ds},$$

$$= -\tfrac{1}{2}\, ds.d.\left(\frac{M}{d^2y}\right) \ldots (13).$$

Erit quoque:

$$v^2 = \frac{Mdx.ds^2}{dx.d^2y} = \frac{Mds^2}{d^2y},\ dt^2 = \frac{dx.d^2y}{Mdx} = \frac{d^2y}{M}.$$

Quod si ex. gr. $R = \alpha D v^2$, atque densitas D constans; habetur:

$$\alpha D.\frac{Mds^2}{d^2y} = -\tfrac{1}{2}\, ds.d.\left(\frac{M}{d^2y}\right), \text{ i.e. } \alpha Dds.\frac{M}{d^2y} = -\tfrac{1}{2}\, d.\left(\frac{M}{d^2y}\right);$$

dividendo per $\left(\frac{M}{d^2y}\right)$, atque integrando, erit:

$$\alpha Ds = \tfrac{1}{2}\, Log.\left(\frac{1}{Cdx^2}\right) - \tfrac{1}{2}\, Log.\left(\frac{M}{d^2y}\right);$$

i. e. $e^{2\alpha Ds} = \frac{d^2y}{CMdx^2}$, vel: $\frac{d^2y}{dx^2} = CM.e^{2\alpha Ds}$.

Ge-

Generatim quidem, in hypothesi: $D = o$, æquatio (13) in hanc:

$$d.\left(\frac{M}{d^2y}\right) = o,$$

abit; unde, integrando, habetur:

$$\frac{M}{d^2y} = \frac{1}{Cdx^2}, \text{ sive: } \frac{d^2y}{dx^2} = CM;$$

quæ quoque æquatio statim prodit, ponendo $D = o$ in allatâ nuperrime:

$$\frac{d^2y}{dx^2} = CM.e^{2\alpha Ds}.$$

Pari prorsus modo operandum, si evanescat M vel N; positis, his in casibus, $dy = const.$ vel $dz = const.$, respective, æquationes (*I*) atque (*II*) æque fiunt simplices, ac in casu nuper allato invenimus.

§. VI.

Hæc, de motibus *in spatio* liberis, in præsenti potentiarum hypothesi consideratis, pro instituti ratione, satis fere attulimus. Superest, ut motus quoque *planos*, alteram motuum liberorum partem tanti momenti, ut per se tractari mereatur, ex generali nostro principio deducere conemur.

mur. Observandum itaque, nos, in præcedente §:o, aliquam quidem quantitatum L, M, N æqualem nihilo posuisse; motum vero quemdam insitum in directione vis illius, quæ evanuit, simul assumsisse. Quod si autem motus in plano in genere definire velis, non solum potentiarum L, M, N quædam evanescat: velocitas quoque projectilis in directione vis evanescentis, æqualis nihilo ponatur. Sic scilicet per se patet, motum in plano fieri; æquationesque (I) & (II), in hoc casu prodeuntes, viam in plano descriptam in genere defi- nient. Ut hæc jam instituto nostro accommodemus, ponatur ex. gr. $N = o$; utque omnis in directione ipsius N motus insitus tollatur, fiat angulus, inter KO (vide fig.) atque planum ABC constitutus, $= o$; i. e. projiciatur corpus in ipso plano abscissarum ABC. Sic primo patet intuitu, corpus in plano ABC necessario motum iri; fietque igitur, hoc in casu:

$$z = o, \; dz = o, \; d^2z = o.$$

Videndum itaque, quales jam prodeant æquationes generales (I) & (II): faciem quidem induent, ab æquationibus (12) & (13) omnino diversam, quamvis res, de qua jam agitur, revera tantum specialior sit casus quæstionis, in fine §:i præcedentis, tractatæ.

Ad æquationem (I) quod attinet, hæc quidem,

dem, in casu præsente, nihil nos docebit; in formam scil. indeterminatam:

$$o = o,$$

ex qua nihil hauriri potest, abitura. Æquatio vero (II), quæ hanc proferet faciem:

$$\varphi\left(D,\ dq\sqrt{\frac{Ldy - Mdx}{dy \,.\, d^2x - dx \,.\, d^2y}}\right) = \frac{Ldx + Mdy}{dq} - \frac{d \,.\, \left\{\frac{(Ldy - Mdx)\, dq^2}{dy \,.\, d^2x - dx \,.\, d^2y}\right\}}{2dq} \quad \ldots\ldots\ldots\ldots (III),$$

sola est, quæ curvam, hoc in casu descriptam, definiat. Neque alia opus esse æquatione, ad trajectoriam plene determinandam, facile apparet. Hæc enim linea, cum omnis in plano ABC jaceat, per æquationem duas inter hoc in plano coordinatas, plane est data. Habemus igitur æquationem (III), projectoriam in genere definientem; quæ quidem, denotantibus D, L, M datas quascumque ipsorum x & y functiones, adeo est universalis, ut nullum non casum motûs liberi plani, resistentiæ vi potentiisque absolutis determinati, complectatur. Ad duas enim vires quascumque L & M, sibi invicem normales, nullam non revocari posse potentiam absolutam in plano ABC agentem, luculenter patet. Hæc igitur æquatio quasi fons atque principium habenda est omnis

de

de motibus in plano liberis doctrinæ, in sequentibus exponendæ.

Ad velocitatem v, tempusque t, quod attinet, valores eorum generales, supra allatos, in præsente casu formam induere:

$$v^2 = \frac{(Ldy - Mdx)dq^2}{dy.d^2x - dx.d^2y},\ dt^2 \left(= \frac{dq^2}{v^2}\right) = \frac{dy.d^2x - dx.d^2y}{Ldy - Mdx}$$

facile perspicitur.

In hypothesi igitur $D = o$, motus in plano liberos (per potentias allati supra generis determinatos), æquatione:

$$2(Ldx + Mdy) = d.\left\{\frac{(Ldy - Mdx)dq^2}{dy.d^2x - dx.d^2y}\right\} \ldots (III'),$$

in genere definitum iri, eadem qua antea ratione evincitur. Neque hoc in casu valores ipsorum v & t nuperrime allatos, aliquo modo commutari, manifestum est.

§. VII.

Motus corporum planos præcipuâ ab Auctoribus curâ consideratos videbis; multaque, quæ huc spectant, præclara, summorum operâ ingeniorum, inventa habentur. Hanc ob caussam nos quoque hanc motuum partem liberorum paullo accuratius examinare, usumque æquationis nostræ (*III*), in casibus quibusdam præcipui momenti illustrandis, monstrare conabimur; sic forsitan aliqua, quæ memoratu sint digna, allaturi.

Quod si igitur primum neque L neque M evanescat, optimorum sane exemplorum copiam adipiscemur, theoriam illam virium, quæ *centripetarum* nomine veniunt, breviter considerando; quam quidem doctrinam, per se late patentem, usûs quoque esse egregii, in confesso est.

Et primum quidem, quod ad *plura* attinet virium centra immobilia (mobilia enim heic non sunt consideranda), in plano ABC collocata, quamvis horum utique contemplatio, ob perplexas, ad quas pervenitur, æquationes, parum habeat utilitatis; attamen, propter facilitatem, qua universalis admodum jam eruitur trajectoriæ æquatio, ad hos primum casus paullisper subsistere non pigebit. Sint igitur a, b, a', b', &c. coordinatæ orthogonales punctorum plurium fixorum, in plano ABC sitorum; r, r', &c. distantiæ eorum variabiles a corpore moto; nec non V, V', &c. (quæ functionum instar ipsorum r, r', &c. considerantur) vires ipsæ centripetæ, ad memorata jam puncta tendentes. Hisce positis, facile patet, in præsente jam casu, haberi:

$L = -$

$$L = -\frac{(x-a)V}{r} - \frac{(x-a')V'}{r'} - \&c.,$$

$$M = -\frac{(y-b)V}{r} - \frac{(y-b')V'}{r'} - \&c.;$$

quibus quidem valoribus, in æquatione nostra (*III*), substitutis, erit utique:

$$\varphi\left(D, \mathrm{d}q\sqrt{\frac{[(y-b)\mathrm{d}x-(x-a)\mathrm{d}y].V}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r} + \frac{[(y-b')\mathrm{d}x-(x-a')\mathrm{d}y]V'}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r'} + \&c.}\right)$$

$$= -\frac{[(x-a)\mathrm{d}x+(y-b)\mathrm{d}y]V}{r\mathrm{d}q} - \frac{[(x-a')\mathrm{d}x+(y-b')\mathrm{d}y]V'}{r'\mathrm{d}q} - \&c$$

$$-\frac{\mathrm{d}.\left\{\frac{[(y-b)\mathrm{d}x-(x-a)\mathrm{d}y]V\mathrm{d}q^2}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r}\right\}}{2\mathrm{d}q}$$

$$-\frac{\mathrm{d}.\left\{\frac{[(y-b')\mathrm{d}x-(x-a')\mathrm{d}y]V'\mathrm{d}q^2}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r'}\right\}}{2\mathrm{d}q} - \&c.$$

$$= -\frac{\mathrm{d}q.\mathrm{d}.\left\{\frac{[(y-b)\mathrm{d}x-(x-a)\mathrm{d}y]^3.V}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r}\right\}}{2[(y-b)\mathrm{d}x-(x-a)\mathrm{d}y]^2}$$

$$-\frac{\mathrm{d}q.\mathrm{d}.\left\{\frac{[(y-b')\mathrm{d}x-(x-a')\mathrm{d}y]^3.V'}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r'}\right\}}{2[(y-b')\mathrm{d}x-(x-a')\mathrm{d}y]^2} - \&c. \quad . \; . \; . \; (14).$$

 Sim-

Simpliciorem induit formam æquatio hæc complicata, positis: $\frac{(y-b)\mathrm{d}x-(x-a)\mathrm{d}y}{\mathrm{d}q}$, $\frac{(y-b')\mathrm{d}x-(x-a')\mathrm{d}y}{\mathrm{d}q}$, &c. (= perpendiculis, ab ipsis virium centris in tangentes trajectoriæ demissis) = u, u', &c. respective; unde prodit:

$$\varphi\left(D,\ \sqrt{V\frac{u\mathrm{d}r}{\mathrm{d}u}+V'\frac{u'\mathrm{d}r'}{\mathrm{d}u'}+\text{\&c.}}\right)$$

$$=-\frac{\mathrm{d}.V\frac{u^3\mathrm{d}r}{\mathrm{d}u}}{2u^2\mathrm{d}q}-\frac{\mathrm{d}.V'\frac{u'^3\mathrm{d}r'}{\mathrm{d}u'}}{2u'^2\mathrm{d}q}-\text{\&c.}\ \ldots\ (15);$$

qua igitur æquatione, minus saltim perplexâ, trajectoria quotcumque descripta centris, in genere definitur. Velocitatem quidem atque tempus, formulis:

$$v^2=\frac{[(y-b)\mathrm{d}x-(x-a)\mathrm{d}y]V\mathrm{d}q^2}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r}+$$

$$\frac{[(y-b')\mathrm{d}x-(x-a')\mathrm{d}y]V'\mathrm{d}q^2}{(\mathrm{d}y.\mathrm{d}^2x-\mathrm{d}x.\mathrm{d}^2y)r'}+\text{\&c.}$$

$$=V\frac{u\mathrm{d}r}{\mathrm{d}u}+V'\frac{u'\mathrm{d}r'}{\mathrm{d}u'}+\text{\&c.};$$

$\mathrm{d}t^2=$

$$dt^2 = \frac{dy\,.\,d^2x - dx\,.\,d^2y}{[(y-b)dx-(x-a)dy]\,\frac{V}{r} + [(y-b')dx-(x-a')dy]\,\frac{V'}{r'} + \&c.}$$

$$= \frac{dq^2}{V\frac{udr}{du} + V'\frac{u'dr'}{du'} + \&c.};$$

in præsente casu definiri, per se facile patet.

Jam vero, sive æquationem (*14*), sive etiam æquationem (*15*), consideraverimus, eas utique indolis adeo intricatæ invenimus, ut, si vel simpliciores tantum casus examinare velimus (pro D, V, V', &c. particulares substituendo functiones), calculo tamen vix ac ne vix quidem aliquid proficere possimus. Excipiatur utique casus, quando $V = \mu r$, $V' = \mu' r'$, &c.; qua scil. hypothesi, vel ipsi, quos attulimus, των L & M valores, utpote in:

$$\mu a + \mu' a' + \&c. - (\mu + \mu' + \&c.)\,.\,x \ \&$$
$$\mu b + \mu' b' + \&c. - (\mu + \mu' + \&c.)\,.\,y$$

abeuntes, luculenter indicant, centra virium quotcumque eumdem prorsus effectum præstitura, ac unicum tantum centrum, cujus coordinatæ sint orthogonales $\frac{\mu a + \mu' a' + \&c.}{\mu + \mu' + \&c.}$ & $\frac{\mu b + \mu' b' + \&c.}{\mu + \mu' + \&c.}$, atque vis centripeta: $(\mu + \mu' + \&c.)\,.\,r_1$, denotante r_1 di-

distantiam ab isthoc virium centro (quod per se quidem memorabile est theorema); at vero, in aliis ipsorum V, V', &c. hypothesibus, neque hæc ad unicum tantum centrum habetur reductio, neque, his in casibus, præter approximationes quasdam, pro re nata, varie instituendas, quidquam fere præstari potest.

Unicum vero centrum virium fixum jamjam paullisper contemplandum; quo in casu æquatio illa (15) in hanc abibit:

$$\varphi\left(D, \sqrt{V\frac{u\mathrm{d}r}{\mathrm{d}u}}\right) = -\frac{\mathrm{d}.V\frac{u^3\mathrm{d}r}{\mathrm{d}u}}{2u^2\mathrm{d}q} = \frac{\sqrt{r^2-u^2}.\mathrm{d}.V\frac{u^3\mathrm{d}r}{\mathrm{d}u}}{2u^2.r\mathrm{d}r} \quad ..(16)$$

(erit scil. $\mathrm{d}q = -\frac{r\mathrm{d}r}{\sqrt{r^2-u^2}}$, decrescente corporis a centro virium distantiâ); velocitas vero atque tempus hos accipiunt valores:

$$v^2 = V\frac{u\mathrm{d}r}{\mathrm{d}u}, \quad \mathrm{d}t^2 = \frac{\mathrm{d}q^2}{V\frac{u\mathrm{d}r}{\mathrm{d}u}}.$$

Positis jam D & V functionibus quibuscumque datis distantiæ r, æquatio nostra (16) adeo utique erit complicata, ut in genere tractari non pos-

possit. Casus etiam speciales non exiguæ sunt difficultatis. Horum primus se offert ille, quo: $R = \alpha D v^2$; qua in hypothesi habebitur:

$$d.V\frac{u^3 dr}{du} = -2u^2 dq.\alpha D V\frac{u dr}{du} = -2\alpha D dq.V\frac{u^3 dr}{du}.$$

Dividendo per $\left(V\frac{u^3 dr}{du}\right)$, atque integrando, fiet:

$$Log.\left(V\frac{u^3 dr}{du}\right) = Log.\,C - 2\alpha\int D dq.$$

Posito igitur, quod $\int D dq$ adeo definiatur, ut evanescat in dato quodam trajectoriæ puncto, ubi $v^2 \left(= V\frac{u dr}{du}\right) = c^2$, nec non $u^2 = k^2$ (quo in loco r in l abituram esse, in sequentibus etiam statuimus); erit utique:

$$Log.\,c^2 k^2 = Log.\,C, \text{ hincque:}$$

$$V\frac{u^3 dr}{du} = \frac{c^2 k^2}{e^{2\alpha\int D dq}} \quad \ldots\ldots \quad (17).$$

Quod si $D = const.$, habebitur:

$$V\frac{u^3 dr}{du} = \frac{c^2 k^2}{e^{2\alpha D q}};$$

eva-

evanescente arcu q, in memorato nuper trajectoriæ puncto.

Observandum quoque est, in casu generali: $R = \alpha D v^m$, æquationem nostram (16) eodem fere, quo nuper, modo tractari posse. Habetur enim:

$$d.\sqrt{\frac{u^3 dr}{du}} = -2u^2 dq.\alpha D \sqrt{}^{\frac{m}{2}} \frac{u^{\frac{m}{2}} dr^{\frac{m}{2}}}{du^{\frac{m}{2}}},$$

$$= -2u^{2-m} dq.\alpha D \left(\sqrt{\frac{u^3 dr}{du}}\right)^{\frac{m}{2}};$$

dividendo per $\left(\sqrt{\frac{u^3 dr}{du}}\right)^{\frac{m}{2}}$, integrando, &c., invenitur:

$$\sqrt{\frac{u^3 dr}{du}} = \left(\int (m-2) u^{2-m} \alpha D dq + Const.\right)^{\frac{2}{2-m}}.$$

Posità $m = 2$, quem supra casum contemplati sumus, formula hæc indeterminatam omnino faciem induet.

Quæ nuper exhibuimus, in æquatione (16) directe tractandà, exemplis, ad ipsas tamen trajectorias cognoscendas, parum profecimus. Hisce igitur missis, ad inversa quædam problemata, quæ hanc spectant materiem, quæque frequenter apud Auctores occurrunt, nos convertamus; quorum ex. gr. sequens est ratio:

Da-